found for the aromatic series exist between the marsh-gas hydrocarbons and the alcohol radicals*. But Fittig's isomeric hydrocarbons belong to the class of metamers having a different chemical structure †, whilst only one kind of structure can be given for the saturated hydrocarbons of the formula $C_n H_{2n+2}$.

I am still pursuing these researches, and hope soon to obtain more definite results.

III. "Introductory Memoir on Plane Stigmatics." By ALEXANDER J. ELLIS, F.R.S. Received March 23, 1865.

(Abstract.)

If from every point in a plane curve parallel straight lines be drawn cutting a given straight line in another series of points, the first set of points, which for convenience may be termed *stigmata*, will be coordinated with the second set of points, which may be termed *indices*, in the same manner as by the system of ordinates and abscissæ in ordinary Cartesian coordinate plane geometry.

Now, the writer remarked that the essence of this coordination consisted in the relation of the two sets of points to each other forming two related figures, and that the circumstances of the ordinates being parallel, and the indices all lying upon one straight line, were accidents. Moreover, he observed that these accidents were not regarded in the ordinary Cartesian equations, where there was nothing to point out that the ordinates were parallel or the abscissæ coincident lines, nor any mention made of the direction of the ordinates and abscissæ. It seemed to him that all the anomalies which occurred in analytical geometry under the name of "imaginaries," were traceable on the one hand to these restrictions in the figure, and on the other to the absence of any indication of their existence in the equations. He therefore thought that it would be possible to generalize plane coordinate geometry as the expression of the law which connects two or more plane figures, point for point, indices with stigmata. These relations would certainly include all those of ordinary geometry, and would, apparently, explain all anomalies hitherto encountered.

It was necessary, in the first place, to form a conception of such a generalized relation between indices and stigmata. Now, in the Cartesian straight line, the lines connecting any three stigmata are proportional to the lines connecting the three corresponding indices, and any pair of the first lines are in the same or opposite directions, according to the relative directions of the corresponding pair of the second lines. If the stigma figure and index figure were no longer straight lines, this could be generalized by saying that the triangle formed by three stigmata was directly

similar to the triangle formed by the three corresponding indices. Again, in the circle referred to rectangular coordinates, the ordinate is a mean proportional between the segments of the diameter to which it is perpendicular, that is, the angle between which segments it bisects. It was easy to generalize this by supposing two lines to be drawn from the index, wherever it might lie on a plane, to the extremities of the same diameter, and the ordinate to bisect the angle between these lines, and to be a mean proportional between their lengths. Other curves were generalized in a similar manner.

It was then necessary to have a notation which should express the relations of both magnitude and direction in one symbol. The ordinary notation was found ill adapted for the purpose. The following was therefore chosen. Capital letters were used to represent geometrical points, and two capital letters to represent a geometrical line in length and direction. The operation of changing one such directed line into another, on the same plane, which the writer had already introduced under the name of *clinant*\*, was represented in the fractional form, the changed line being written below and the other above, but instead of capital letters the corresponding small letters were employed, to show that we were dealing with operations and not with quantities; and when the changed line was the axis of reference itself, it was not expressed. The notation thus introduced closely simulates that employed in M. Chasles's ' Géométrie Supérieure,' but it is totally different in principle. It has the advantage of clearly showing the geometrical operation indicated by each algebraical change, and of perfectly obeying the laws of ordinary algebra, while it not only generalizes but frequently abridges the operations of analysis. By means of these clinants it became easy to express the relations between the stigma figure and index figure by equations which are of exactly the same character as the Cartesian equations, and from which the latter, with all their results, can be strictly deduced.

In the present introductory memoir the writer has confined himself to the investigations connected with the stigmatic straight line, explaining its equation and direction, the intersections of two such lines, the angles between them, and their distances from stigmatic points. These preliminary propositions being given with the requisite detail and illustrated by deducing from them the ordinary Cartesian formulæ, the rest of the memoir is occupied with the generalization of the fundamental theories necessary for the successful application of the stigmatic theory to plane geometry ; such as those relating to the stigmatic triangle, an harmonic ratio of geometrical points anywhere situate on a plane and of stigmatic rays, pencils of such rays with their homography and involution, and the complete

---

\* " On the Laws of Operation, and the Systematization of Mathematics," ' Proceedings,' May 26, 1859, vol. x. p. 89, at bottom. " On Scalar and Clinant Algebraical Coordinate Geometry," *ibid.* March 22, 1860, vol. x. p. 420. The notation in the present memoir is new.

quadrilateral. Then the nature of the change of coordination, by which a new index figure is coordinated with the same stigma figure, is explained, and bilinear and directional coordination introduced and illustrated by applying them to deduce the usual formulæ for the transformation of Cartesian coordinates from oblique to oblique, and from oblique to polar. This is followed by the most general theory of transversals cutting or inter-secting upon any stigmatic curve, and by trilinear coordination. The equation to a stigmatic point is then discussed, giving rise to classes of stigmatic curves with bipunctual and tripunctual coordination. The investigations on trilinear and tripunctual coordination contain generaliza-tions of Professor Plücker's 'Point and Line Coordinates,' by which their precise geometrical meaning, even when "imaginary," and even in more general cases than those "imaginaries" which he contemplated, becomes manifest from the very form of the equations.

Although details have been avoided as much as possible in the latter part of the memoir, the writer hopes that sufficient has been given to enable any mathematician to apply the theory with ease and safety to the gene-ralization and linear realization of every theory on plane geometry which has hitherto been propounded. The conception is equally applicable to solid geometry, but will there require the algebra of quaternions, which, being non-commutative, establishes a well-marked line of separation between plane and solid stigmatics. The writer has not found a trace of this gene-ralization in the works of any previous author, but the relations, when pointed out, appear too obvious to have escaped all notice hitherto. The writer believes that in any case no complete theory, such as that presented in this memoir, has been previously founded upon any similar conception.

The Society then adjourned, over the Easter Recess, to Thursday, April 27.

*April* 27, 1865.

Major-General SABINE, President, in the Chair.

Pursuant to notice given at the last Meeting, Sir Henry Holland pro-posed, and Dr. Bence Jones seconded, His Royal Highness the Count of Paris for election and immediate ballot.

The ballot having been taken, His Royal Highness the Count of Paris was declared duly elected.

The following communications were read :—

I. " Further Experiments on the Production of Organisms in Closed Vessels." By GEORGE CHILD, M.D. Communicated by Pro-fessor PHILLIPS. Received March 30, 1865.

The researches, an account of which is contained in the following paper, are in continuation of those which, through the kindness of Prof. Phillips,